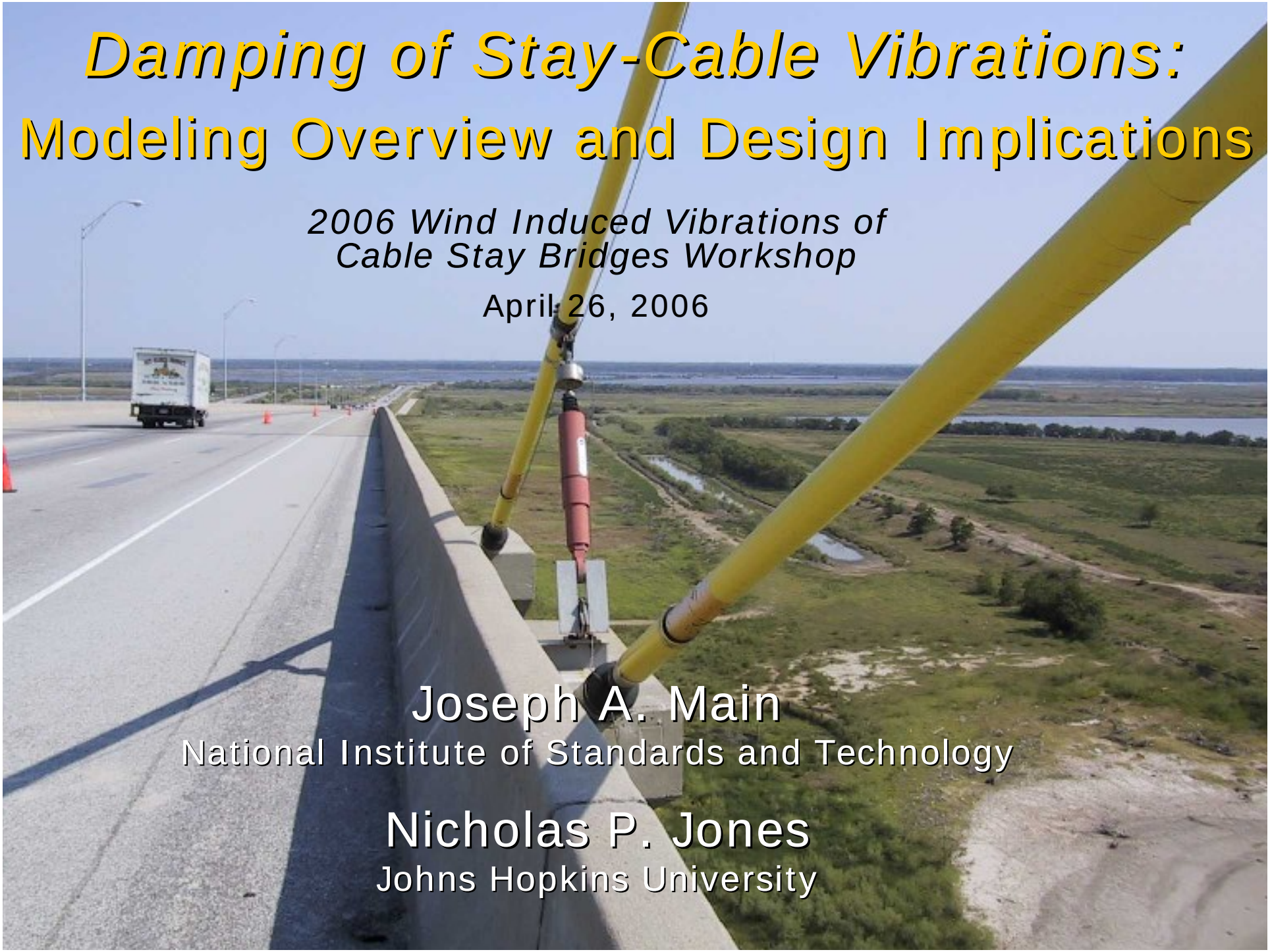


Damping of Stay-Cable Vibrations:
Modeling Overview and Design Implications

2006 Wind Induced Vibrations of Cable Stay Bridges Workshop

April 26, 2006

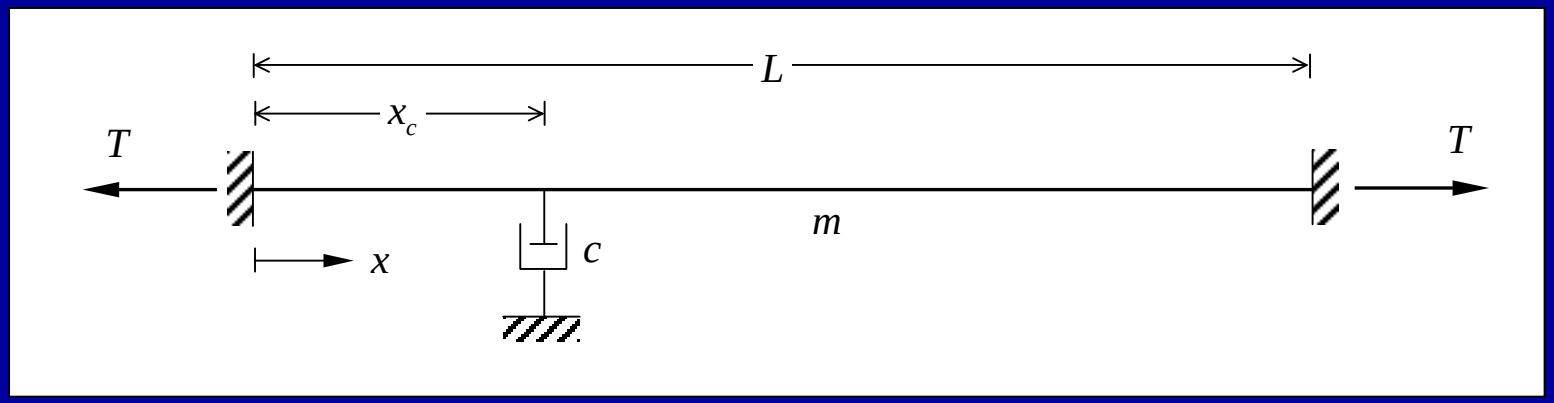
Joseph A. Main
National Institute of Standards and Technology

Nicholas P. Jones
Johns Hopkins University

Outline

- Taut string with viscous damper
- Influence of bushings at anchorage
- Influence of bending stiffness
- Influence of damper nonlinearities
- Concluding remarks

Taut String with Viscous Damper



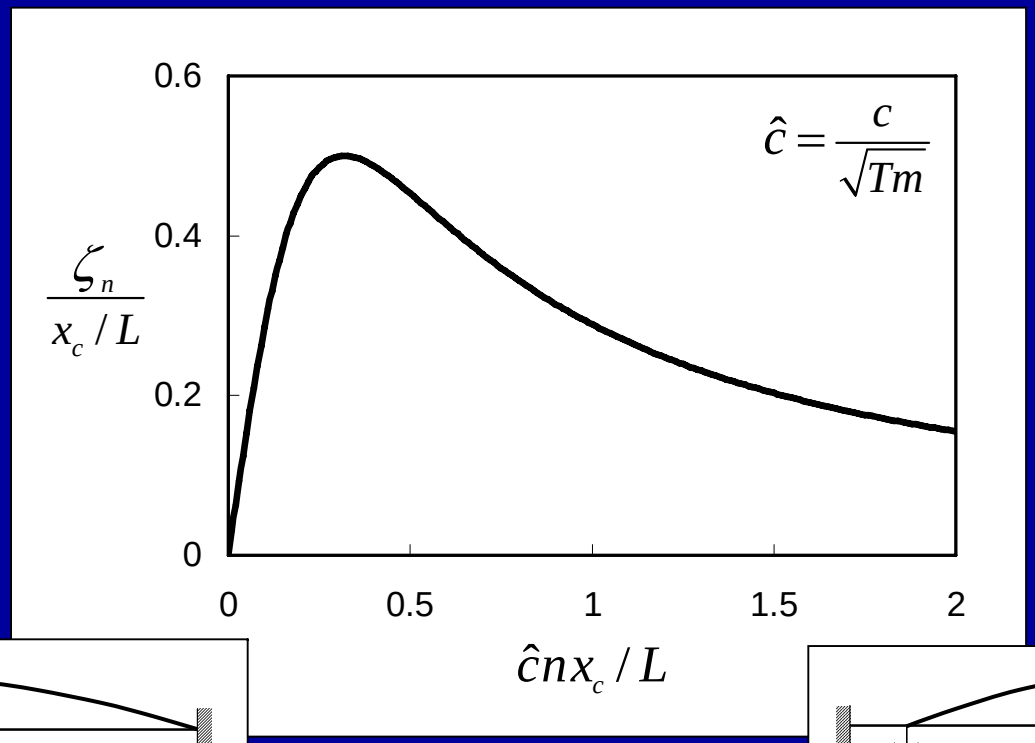
<u>Taut string model:</u>

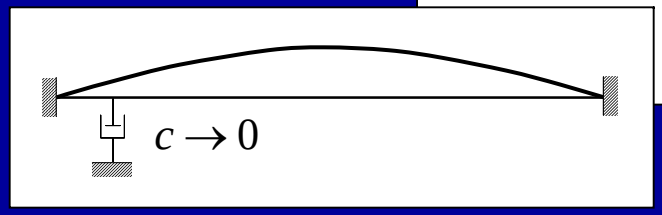
- Neglects flexural stiffness: “kink” at damper
- Neglects sagged profile, axial extensibility
- Reasonable approximation in many cases

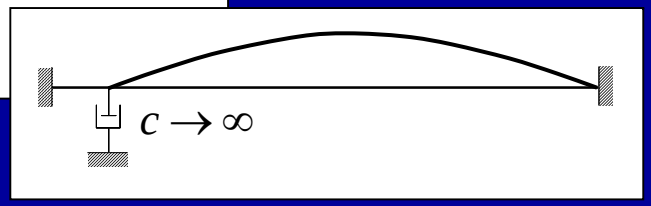
“Universal Estimation Curve”

(Pacheco, Fujino, and Sulekh 1993)

- Identified numerically
- Relates modal damping ratios to viscous damper coefficient
- Useful in damper design for stay cables

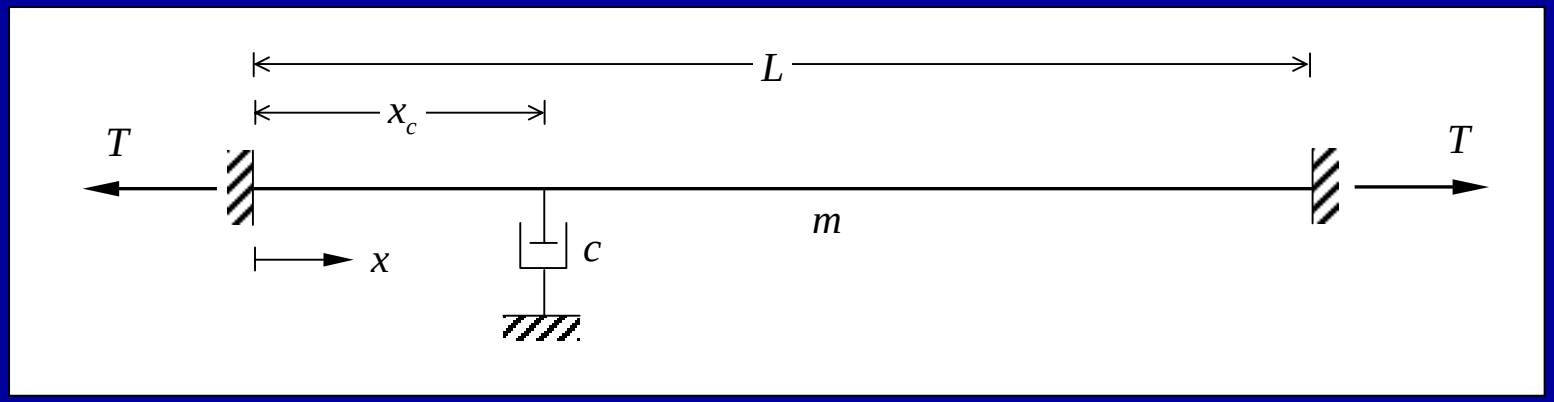






Analytical Solution for Taut String

(Krenk 2000, Main and Jones 2002)



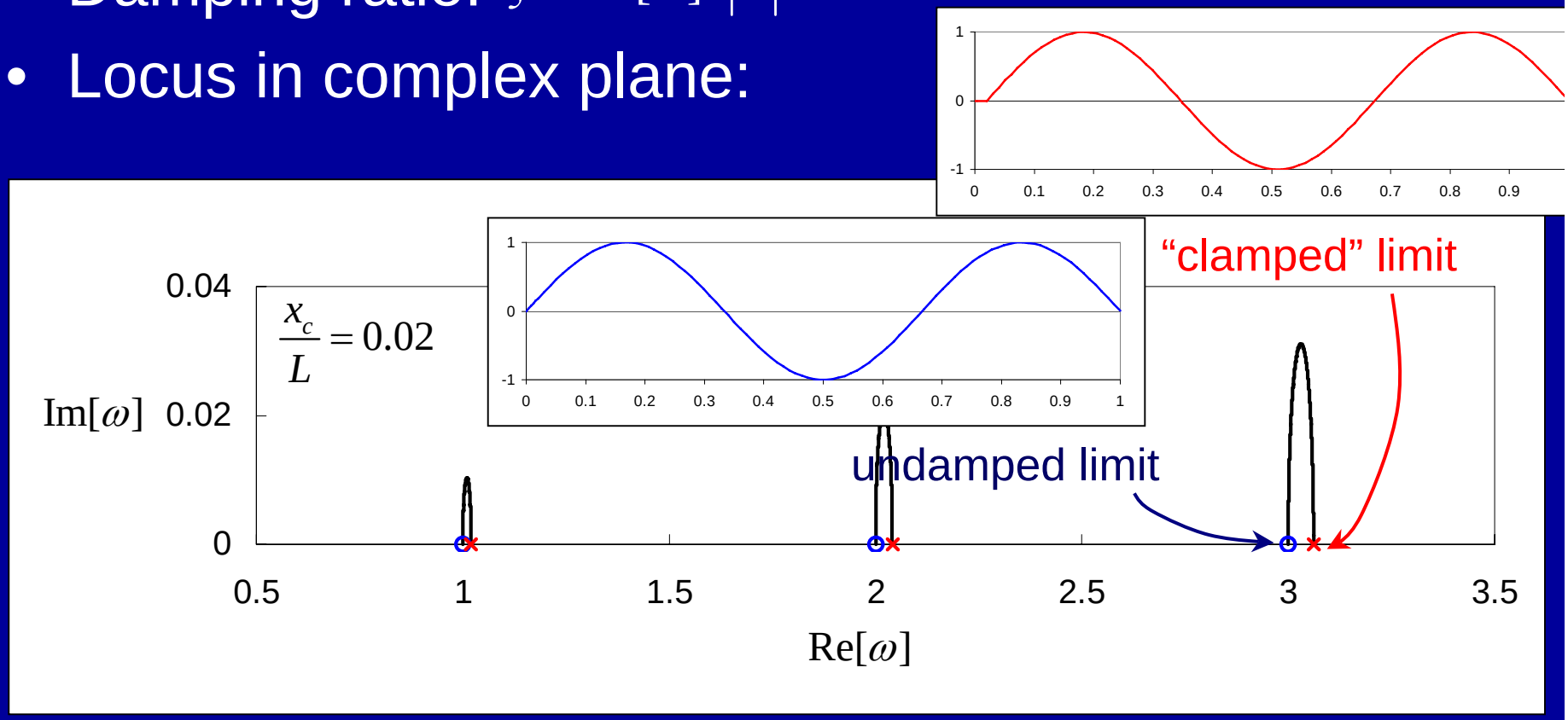
- PDE over two segments: $m\ddot{y} = Ty''$
- Separation of variables: $y(x,t) = Y(x)e^{i\Omega\tau}$
- BC's and continuity: $y(0) = 0,\ \ y(L) = 0,\ \ y(x_c^+) = y(x_c^-)$
- Force balance at damper: $T[y'(x_c^+) - y'(x_c^-)] = c\dot{y}(x_c)$

➔ Transcendental equation for eigenfrequencies:

$$\boxed{\sin(\pi\omega) + i\frac{c}{\sqrt{Tm}}\sin\left(\pi\omega\frac{x_c}{L}\right)\sin\left(\pi\omega\frac{L - x_c}{L}\right) = 0} \qquad \omega = \Omega\left(\frac{\pi}{L}\sqrt{\frac{T}{m}}\right)^{-1}$$

Complex-Valued Eigenfrequencies

- Real part: *damped frequency*
- Imaginary part: *decay rate*
- Damping ratio: $\zeta = \mathrm{Im}[\omega]/|\omega|$
- Locus in complex plane:



Asymptotic Approximations

Complex-valued frequency increment: $\Delta\omega_n = \omega_n - \omega_{0n}$

(assumed small)

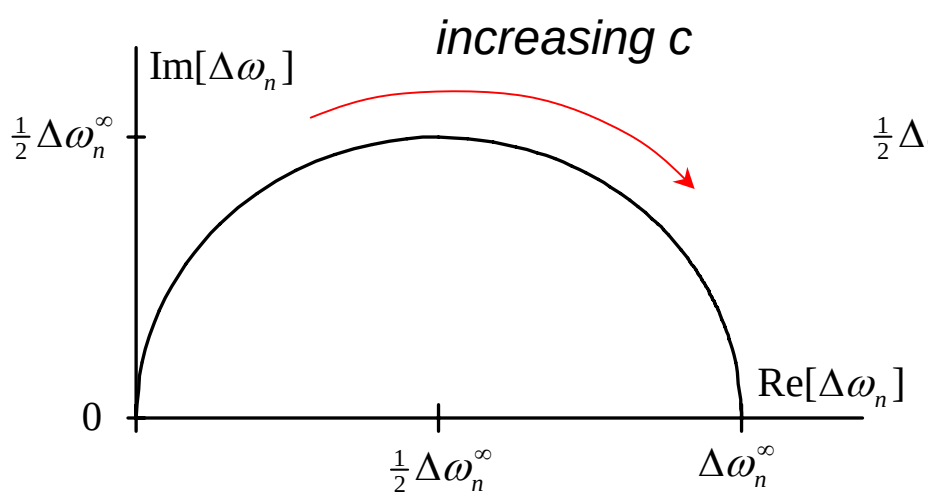
Asymptotic approximation:

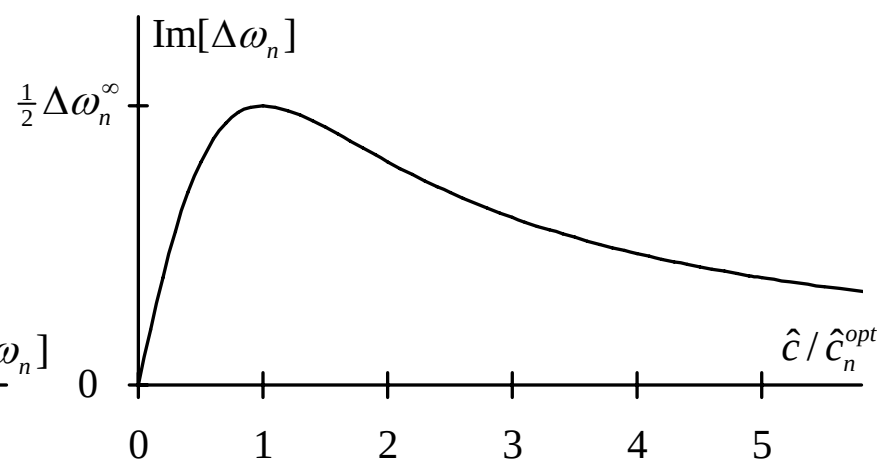
$$\boxed{\Delta\omega_n \cong \Delta\omega_{\infty n} \frac{i\hat{c}/\hat{c}_n^{opt}}{1 + i\hat{c}/\hat{c}_n^{opt}}}$$

where $\hat{c}_n^{opt} \cong \pi n x_c / L$

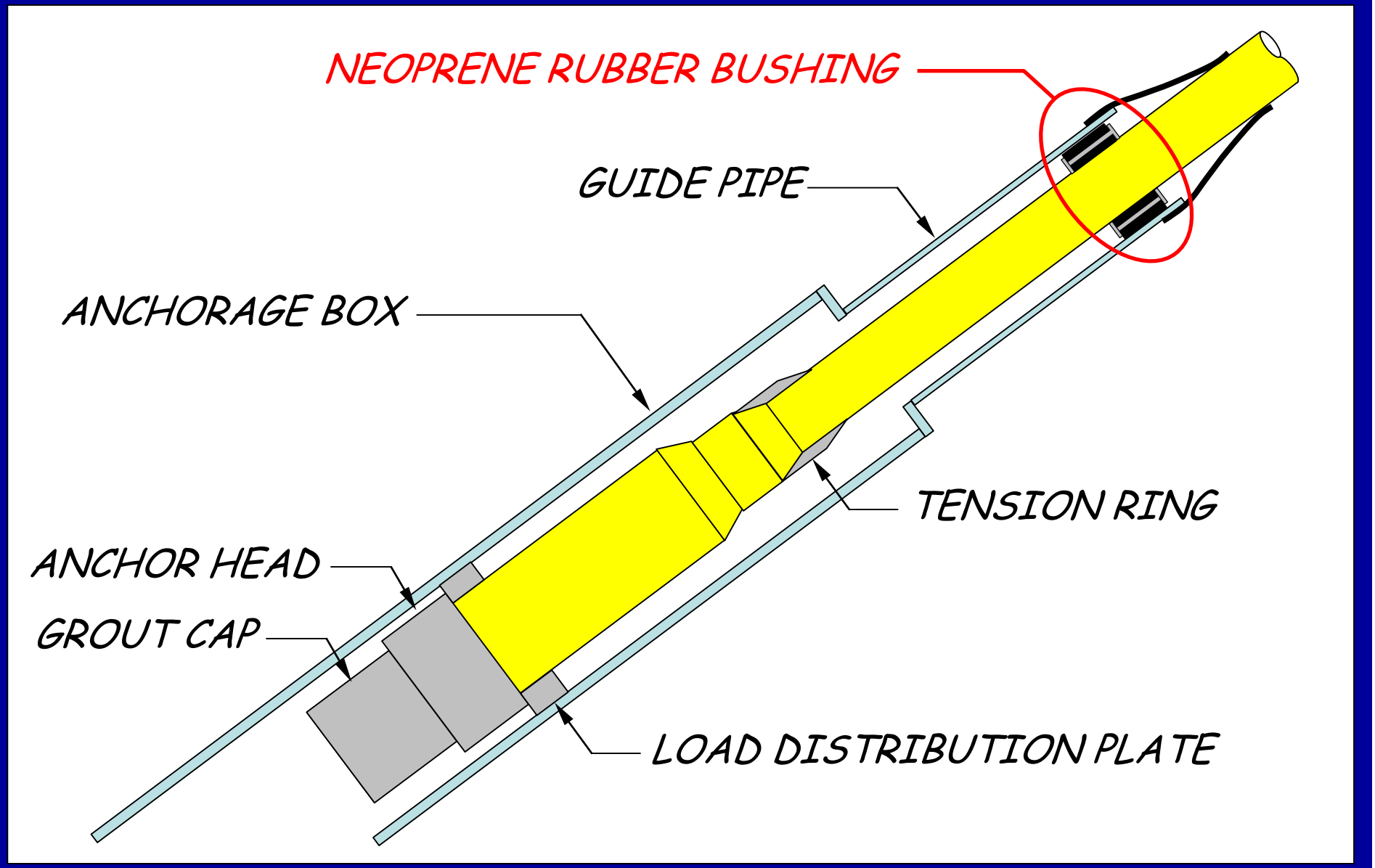
$\Delta\omega_{\infty n} \cong n x_c / L$

<u>Semi-Circular Locus</u>





Influence of Bushings at Anchorage

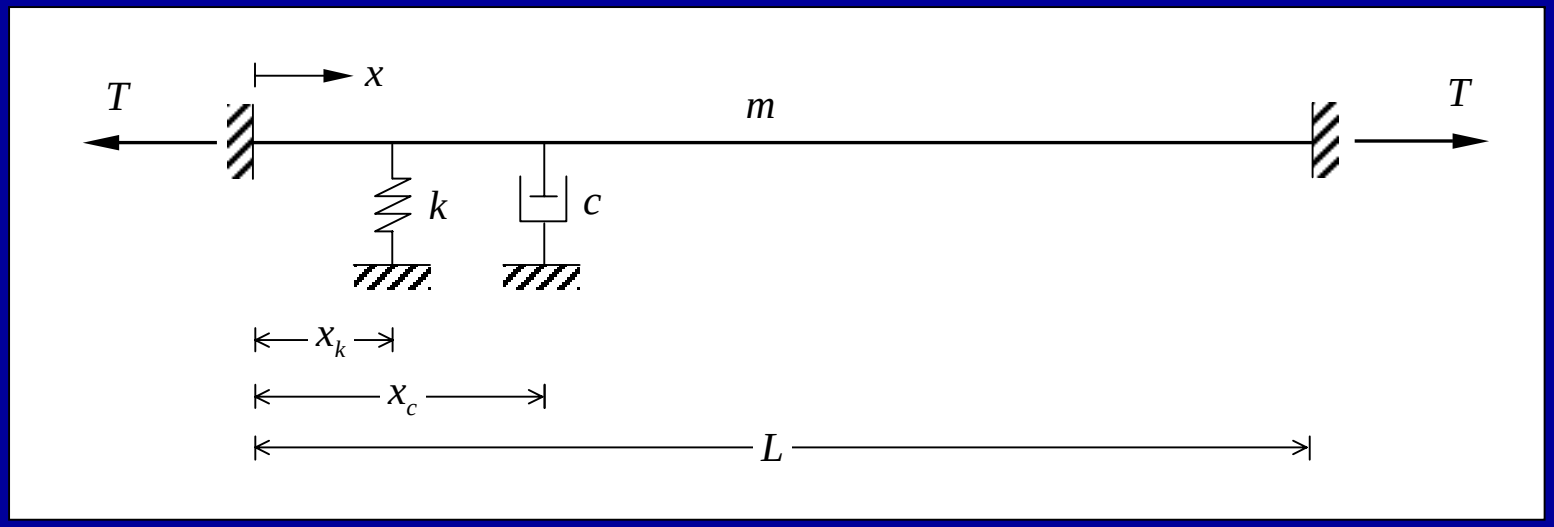


Bushing Removal: Stay with Damper

Fred Hartman Bridge

(Houston, Texas)

Analytical Model: Damper and Bushing

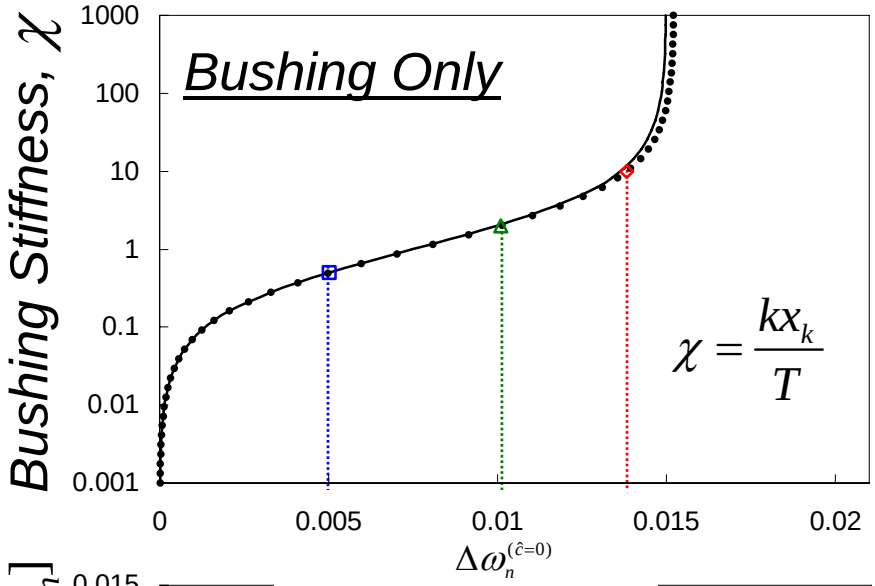


- Cable: taut string $m\ddot{y} = Ty''$
- Damper: linear dashpot $T[y'(x_c^+) - y'(x_c^-)] = c\dot{y}(x_c)$
- Bushing: linear spring $T[y'(x_k^+) - y'(x_k^-)] = ky(x_k)$
- Boundary conditions, continuity at damper

➔Transcendental equation for complex eigenfrequencies

  – Efficient iterative solution scheme
  – Explicit asymptotic approximate solution

Complex-Valued Eigenfrequencies



Asymptotic: ——

Exact:

$\hat{c} = 0$ •

$\chi = 0$ ○

$\chi = 0.5$ □

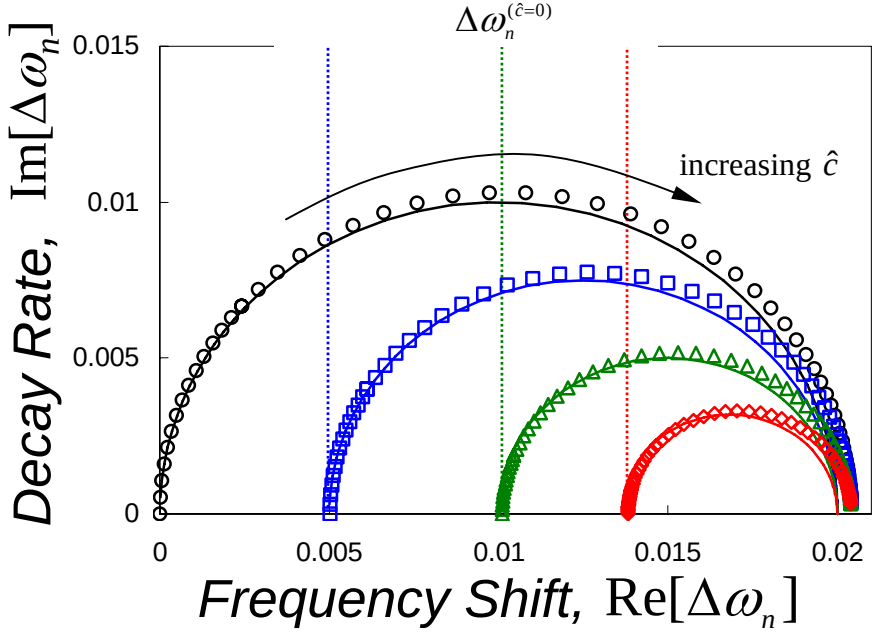
$\chi = 2$ △

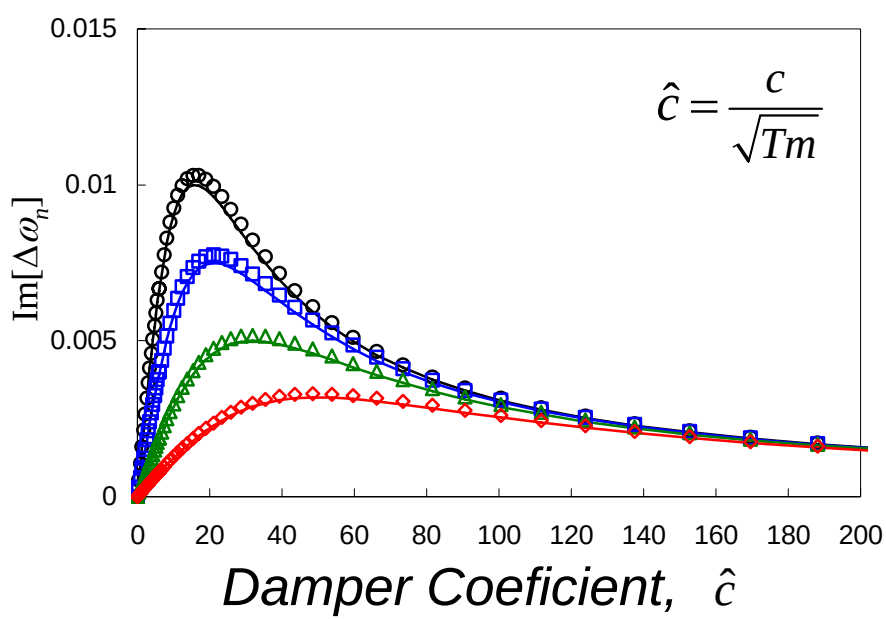
$\chi = 10$ ◇

$n = 1$

$x_k / x_c = 0.75$

$x_c / L = 0.02$





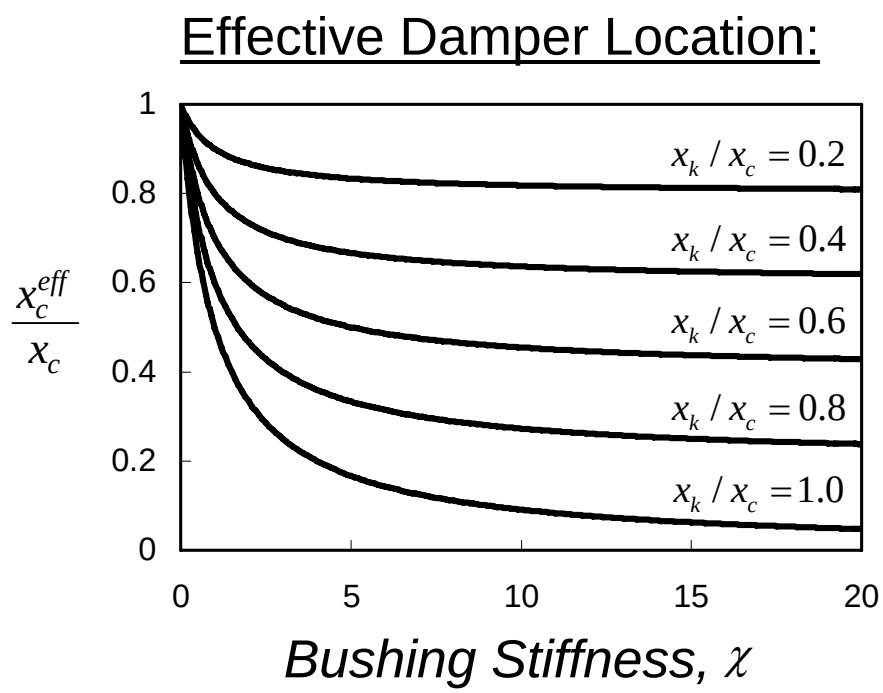
Generalization of "Universal Curve"

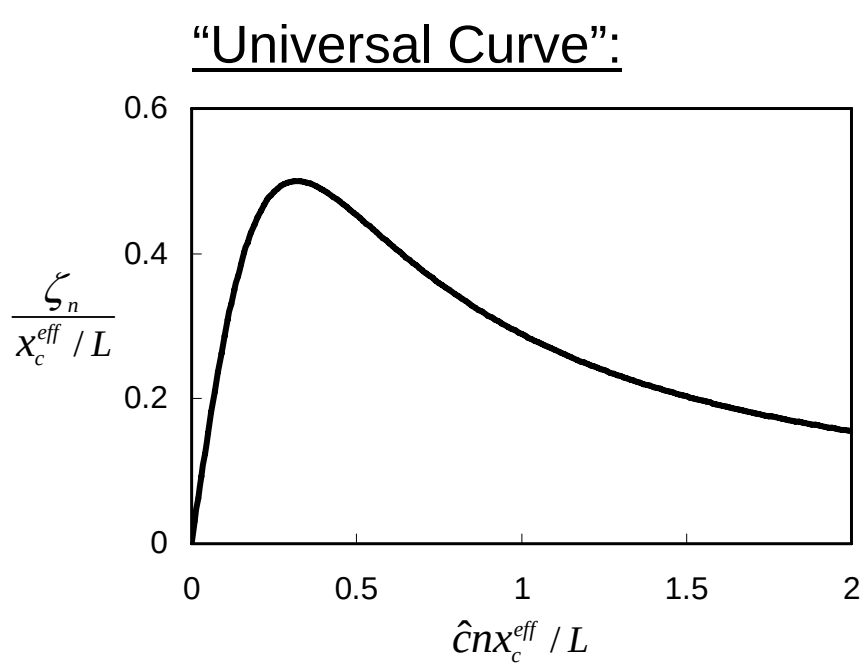
- Effective damper location:

$$\frac{x_c^{eff}}{x_c} = \frac{1+\chi(1-x_k/x_c)}{1+\chi}$$

- Generalized "universal curve":

$$\frac{\zeta_n}{x_c^{eff}/L} \cong \frac{\pi \hat{c} n x_c^{eff}/L}{1+(\pi \hat{c} n x_c^{eff}/L)^2}$$





Effects of Bushing Stiffness

Reduction of x_c^{eff} leads to:

- Reduction of attainable damping ratios:

$$\zeta_n^{opt} \cong \frac{1}{2}\frac{x_c^{eff}}{L}$$

- Increase of optimal damper coefficient:

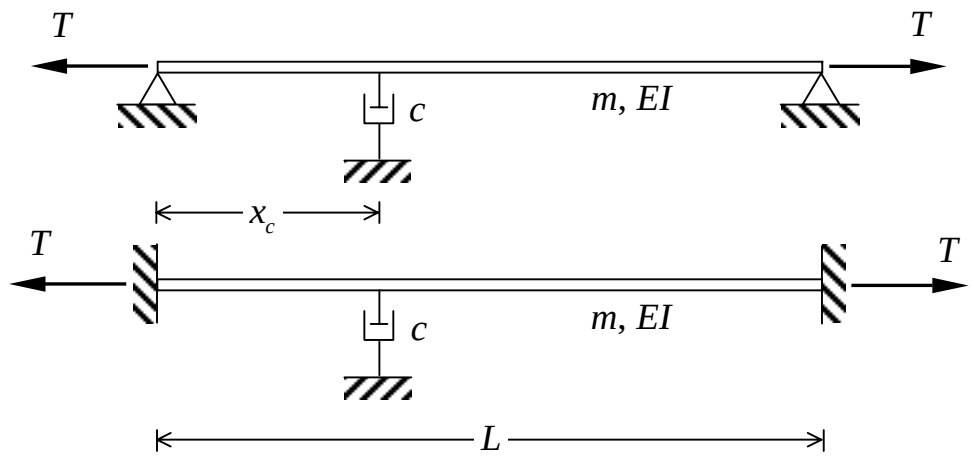
$$c_n^{opt} \cong \frac{\sqrt{Tm}}{\pi n x_c^{eff} / L}$$

Influence of Bending Stiffness

- Tensioned beam with viscous damper:

Pinned supports:

Clamped supports:



nondimensional parameter:

$$\gamma = \sqrt{\frac{TL^2}{EI}}$$

most stay cables:

$$10 < \gamma < 600$$

- PDE over two segments: $EIy'''' - Ty'' + m\ddot{y} = 0$
- Force balance at damper: $EI[y'''(x_c^-) - y'''(x_c^+)] = c\dot{y}(x_c)$
- Boundary conditions, continuity at damper

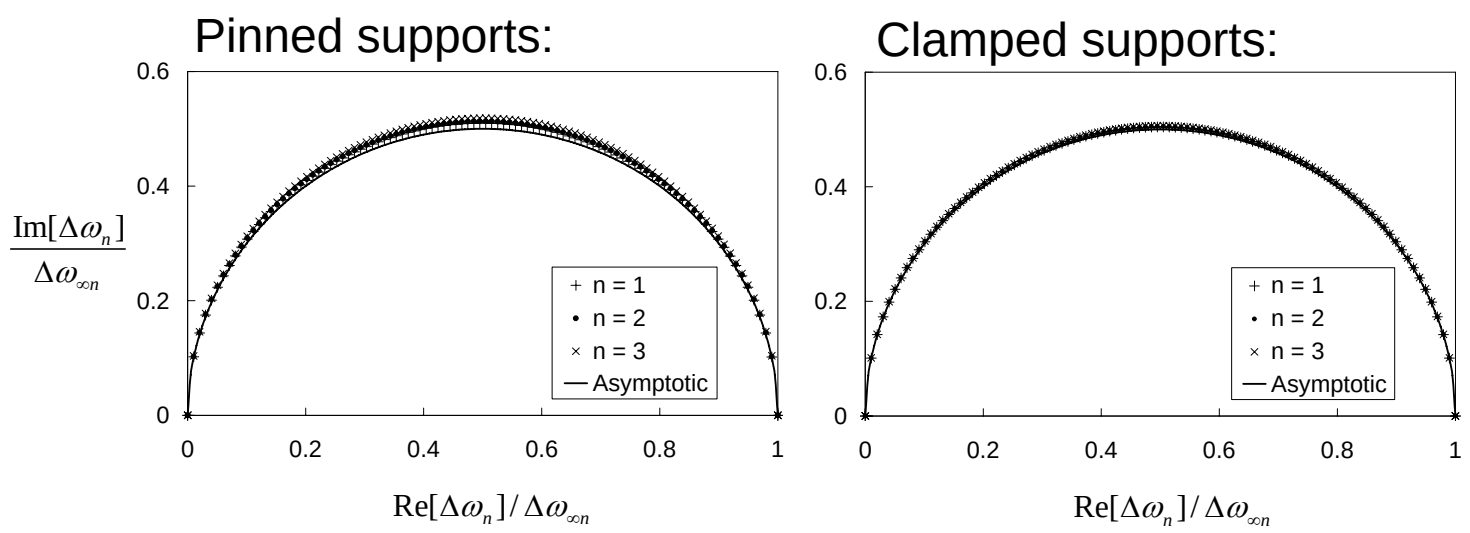
➔ Transcendental equations for complex eigenfrequencies

  – Efficient iterative solution schemes
  – Explicit asymptotic approximate solutions

Asymptotic Approximations

Same form as for taut string!

$$\Delta\omega_n \cong \Delta\omega_{\infty n} \frac{i\hat{c}/\hat{c}_n^{opt}}{1+i\hat{c}/\hat{c}_n^{opt}}$$



$\gamma = 50$

$x_c / L = 0.02$

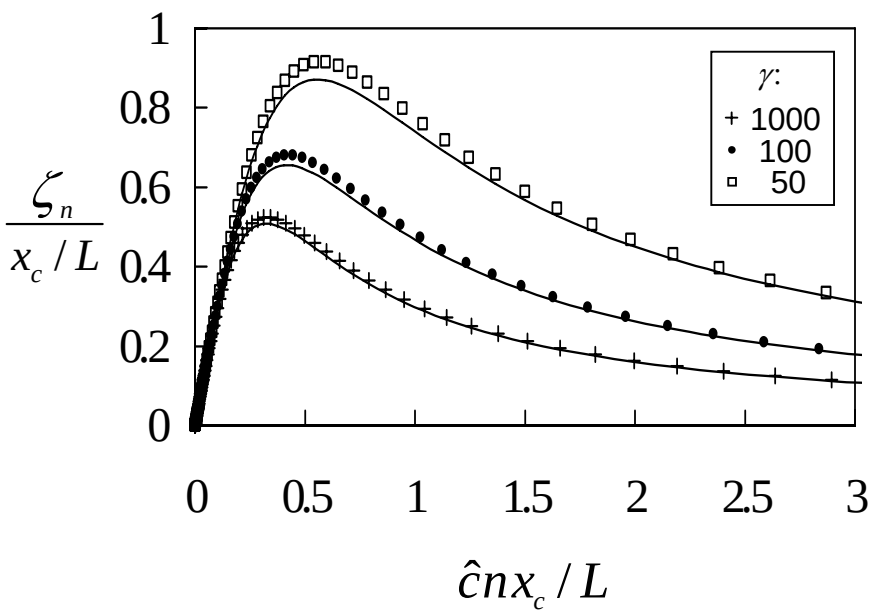
Expressions for $\Delta\omega_{\infty n}$ and $\hat{c}_n^{opt}$ depend on support conditions

Evolution of “Universal Curve”

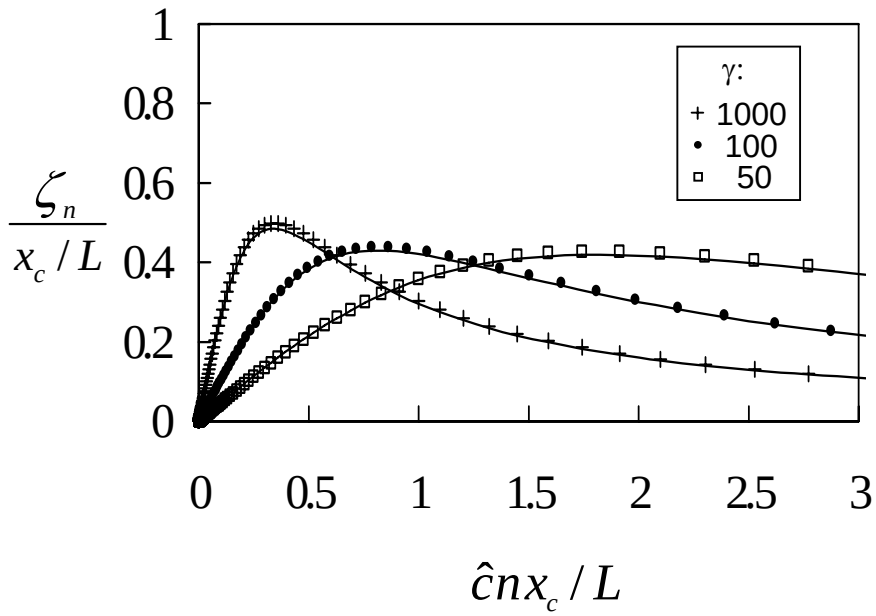
Shifting of curve depends on support conditions:

$x_c / L = 0.02$

Pinned supports:



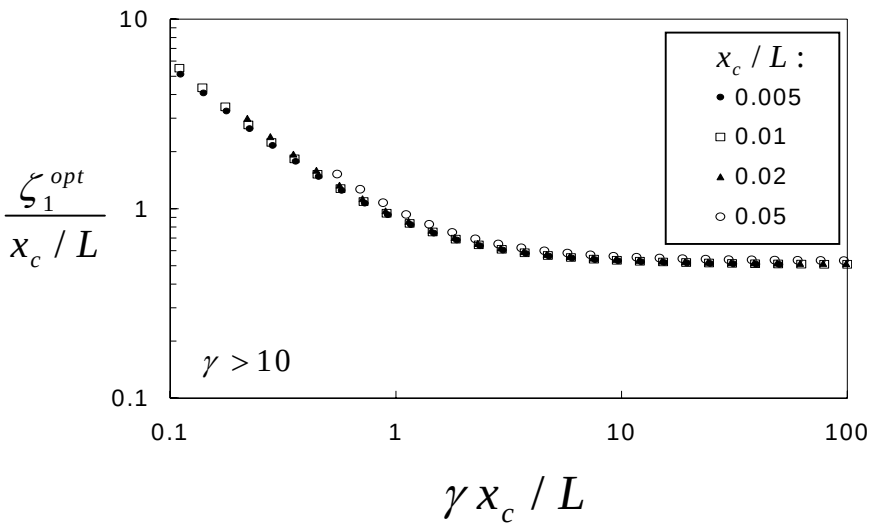
Clamped supports:



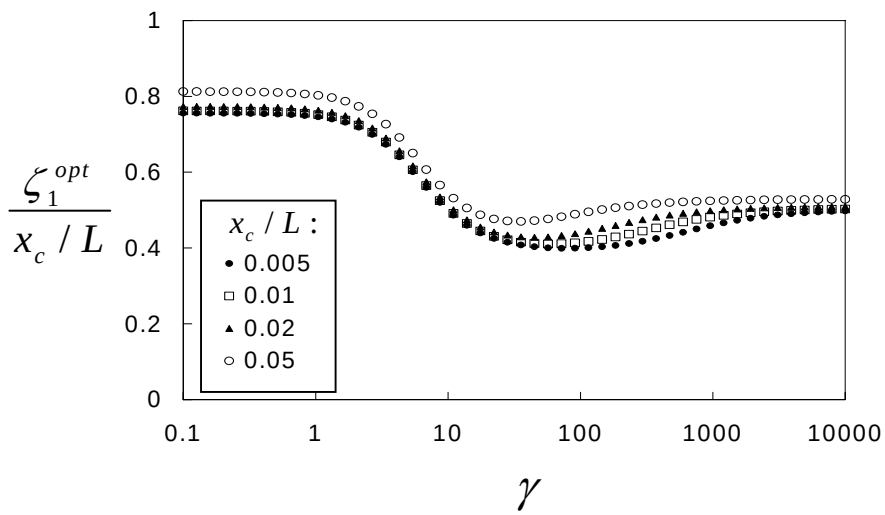
Optimal Damping Ratios

- Larger frequency shifts for pinned supports
  ➔ larger damping

Pinned supports:

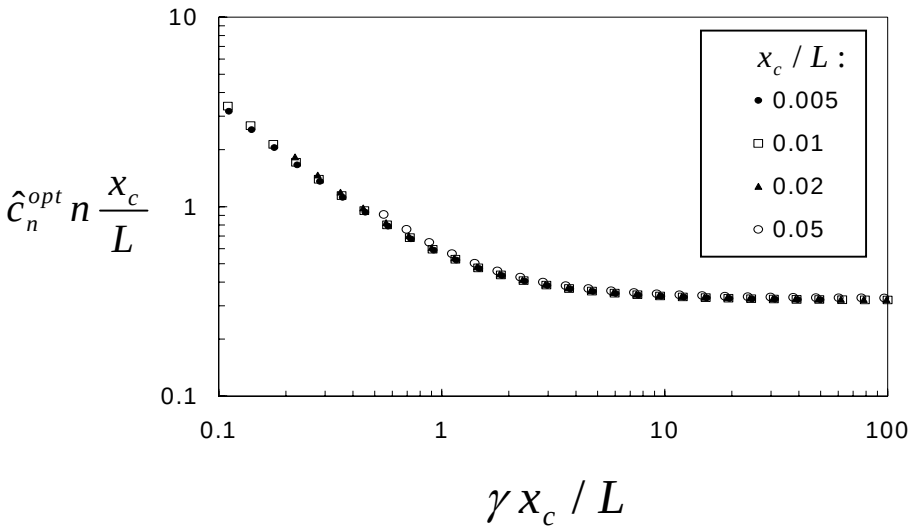


Clamped supports:

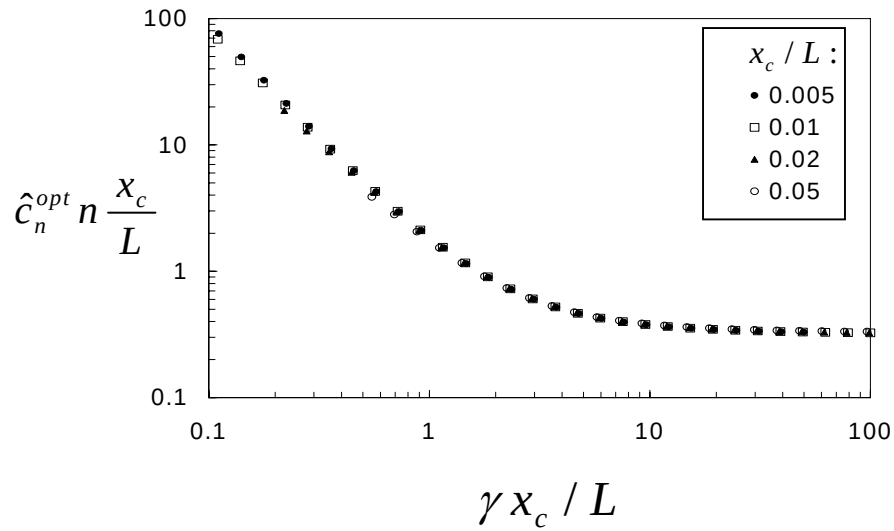


Optimal Damper Coefficient

Pinned supports:



Clamped supports:

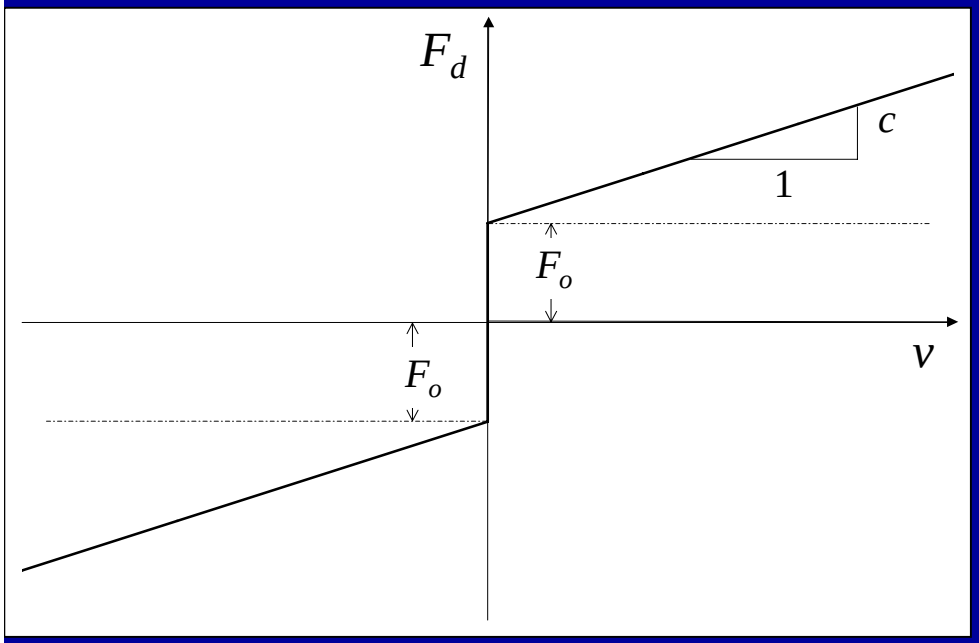


- Significant increases over “taut string” values (currently used in design)
- Much stronger increase for clamped supports
- Within 20% of taut string values for $\gamma x_c / L > 10$

Damper Nonlinearities

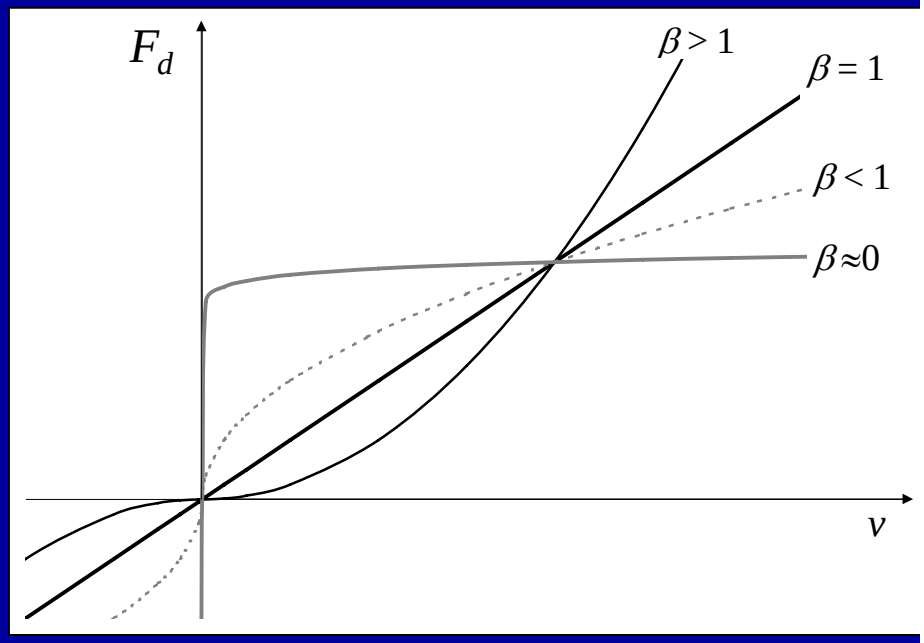
Friction/Viscous Damper:

$$F_d(v) = F_o \operatorname{sgn}(v) + c \cdot v$$



Power-Law Damper:

$$F_d(v) = c|v|^{\beta} \operatorname{sgn}(v)$$



Equivalent Viscous Solutions

- Compute coefficient of "equivalent viscous damper" based on energy dissipation:

$$c_{eq} = \frac{\Delta W_d}{\pi \alpha^2 \omega}$$

- Equivalent viscous coefficient depends on frequency of oscillation ω and amplitude at damper α

<u>Friction/Viscous Damper</u>

$$c_{eq} = \frac{4F_o}{\pi \omega \alpha} + c$$

<u>Power-Law Damper</u>

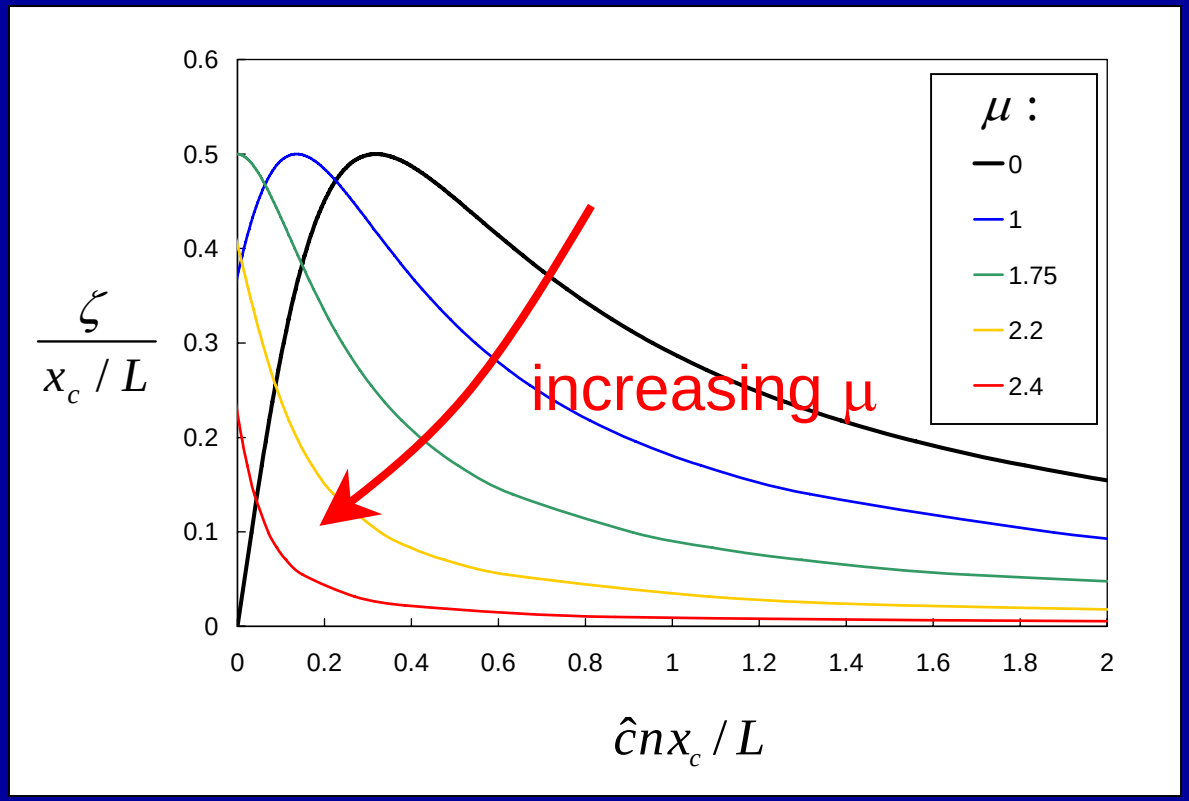
$$c_{eq} = c(\omega\alpha)^{\beta-1} f(\beta)$$

where $f(\beta) = \frac{2}{\sqrt{\pi}} \frac{\Gamma(1+\beta/2)}{\Gamma(3/2+\beta/2)}$

- Inaccurate for strong nonlinearities (e.g., frictional "stick-slip") → efficient numerical solution scheme developed

Friction/Viscous Damper

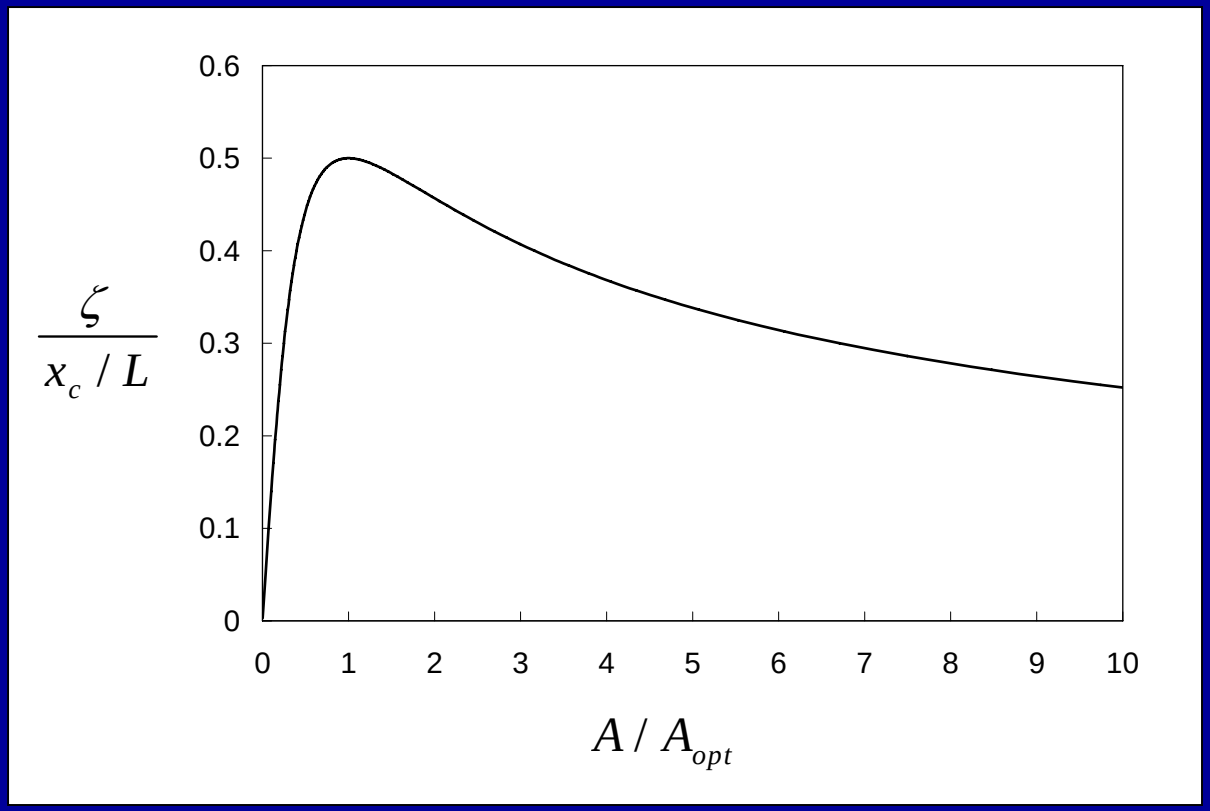
- Nondimensional friction/amplitude parameter: $\mu = \frac{F_0}{T}\left(\frac{An}{L}\right)^{-1}$
- Influence on "Universal Curve":
  - Reductions in optimal value of viscous coefficient
  - Eventually, reductions in attainable damping ratios



Square-Root Damper ($\beta = 0.5$)

- Damping independent of mode number

  Advantage: damp multiple modes
- Damping is a function of peak modal amplitude, A
- Optimal amplitude can be designed (e.g., set $A_{opt} = 1.5D$)



Concluding Remarks

- Importance of damper-induced frequency shifts
- Persistence of semi-circular locus and general form of "universal curve"
- Optimal damper coefficient and maximum attainable damping ratios affected by bushings, bending stiffness
- Asymptotic approximations obtained for many cases of practical interest: efficient and useful in design
- "Equivalent viscous" approach used to extend linear solutions to nonlinear dampers
  - Friction can reduce optimal damper coefficient and damping ratios → *should be accounted for in design*
  - Damping of square-root damper independent of mode number

Acknowledgements

- TXDOT
- FHWA
- NSF
- Jack Spangler, senior mechanical technician, JHU